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ELECTROWEAK PROCESSES AND PRECISION TESTS¹

Probir Roy
Tata Institute of Fundamental Research
Homi Bhabha Road, Bombay 400 005
India

CONTENTS

- Review of the electroweak sector of the minimal standard model
- Basic LEP processes at the tree level
- 1-loop radiative corrections in the on-shell renormalization scheme
- Star scheme and oblique corrections
- Introduction to oblique parameters

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- **Review of the electroweak sector of the Minimal Standard Model**

The electroweak theory of Glashow-Weinberg-Salam [1] and the quantum chromodynamic [2] description of strong interactions together comprise the Standard Model (SM) of particle physics. The interactions of this theory are derived by use of the local gauge principle based on the gauge group structure

Fig. 1. Gauge group structure of SM

The Higgs mechanism [3] screens out a part of the electroweak forces which then constitutes the short-range weak interactions. Left residually is an exact abelian long-range quantum electrodynamic interaction characterized by a conserved electromagnetic charge Q^{EM} . The Higgs mechanism does not touch the QCD part which remains an exact confining $SU(3)$ gauge theory. We shall not deal with this sector, leaving it to the lectures of D.P. Roy.

Let us first remark on the behavior of three generations of matter particles/fields with respect to these gauge groups. $SU(3)_C$ recognizes the fundamental color triplet of quarks for each of six flavors and treats left- and right-chiral ones on equal footing leaving all neutrinos and charged leptons as color singlets. $SU(2)_L$ distinguishes between flavor doublets of left-chiral fermions $f_L = \frac{1}{2}(1 - \gamma_5)f$ with $T_{3L} = +\frac{1}{2}$ (up type), $-\frac{1}{2}$ (down type) and singlets of right-chiral fermions $f_R = \frac{1}{2}(1 + \gamma_5)f$ with $T_{3R} \equiv 0$. Of the twelve different fermions, eleven have values or upper limits on their masses [4] but the twelfth, namely the top quark, only has a lower limit so far on its mass – in the vicinity of 113 GeV. The remaining factor group $U(1)_Y$ attributes a weak hypercharge Y to each chiral fermion which is, in general, different for the L - and R -components. Y takes values given by the weak Gell-Mann-Nishijima formula

$$Q = T_{3L} + \frac{1}{2}Y,$$

with $Q_\nu = 0$, $Q_{e,\mu,\tau} = -1$, $Q_{u,c,t} = \frac{2}{3}$, $Q_{d,s,b} = -\frac{1}{3}$ in units of the positron charge. The corresponding properties of antiparticles can be simply obtained by C -conjugation.

$$\begin{aligned} & \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} u \\ d \end{pmatrix}_L; e_R, u_R, d_R. \\ & \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L; \mu_R, c_R, s_R. \\ & \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L; t_R, b_R, \tau_R. \end{aligned}$$

Fig. 2. Fermion content of SM.

Local gauge invariance – in the absence of any spontaneous symmetry breakdown – makes the following requirement. To every generator of each factor group of G_{SM} , there must correspond a massless gauge boson coupling minimally to matter fields; furthermore, all gauge bosons in a simple factor group must have a universal coupling strength. In the electroweak sector they are W_μ^a ($a = 1, 2, 3$) and B_μ with $SU(2)$ and $U(1)$ gauge coupling strengths g and g' respectively. The fermion-gauge interactions are then included and specified in the generalized fermion kinetic energy terms

$$\mathcal{L}_{fg} = i \sum_{f,a} \left[\bar{f}_L \gamma^\mu \left(\partial_\mu - ig W_\mu^a T^a - ig' B_\mu \frac{Y}{2} \right) f_L + \bar{f}_R \gamma^\mu \left(\partial_\mu - ig' B_\mu \frac{Y}{2} \right) f_R \right] \quad (1)$$

in the Lagrangian density. In (1) f is a generic fermion, a sums over 1 to 3 and $T^a = \frac{1}{2} \tau^a$. The generalized gauge boson kinetic energy terms are

$$\mathcal{L}_{gb} = -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4}(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a + ig \epsilon^{abc} W_\mu^b W_\nu^c)^2. \quad (2)$$

The Higgs mechanism generates masses for the weak bosons via the spontaneous breakdown of $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$. This is done in

the Minimal Standard Model (MSM) in a certain way and that is perhaps where the physics that lies beyond is most likely to show up. The procedure is to introduce a complex doublet Higgs scalar field ϕ with $Y = 1$:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \phi^c = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix}.$$

This leads to a gauge-Higgs term as a generalized Higgs kinetic energy:

$$\mathcal{L}_{gh} = \sum_a \left| \left(\partial_\mu - igW_\mu^a T^a - ig'B_\mu \frac{Y}{2} \right) \phi \right|^2. \quad (3)$$

Furthermore, gauge invariant and renormalizable quadratic and quartic self-Higgs terms

$$\mathcal{L}_{hh} = -V(\phi) \equiv \mu^2 |\phi|^2 - \lambda |\phi|^4, \quad (4)$$

with $\lambda, \mu^2 > 0$ are postulated. The minimization of $V(\phi)$ makes ϕ acquire a vacuum expectation value (VEV) which is real (since any phase can be rotated away):

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad \langle \phi^c \rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}. \quad (5)$$

The introduction of the VEV (5) into (3) leads to the gauge boson mass terms

$$\mathcal{L}_M = -\frac{1}{2} \left[\frac{1}{2}(gv) \right]^2 \left[(W_\mu^1)^2 + (W_\mu^2)^2 \right] - \frac{1}{2} v^2 \left[\frac{1}{2}(gW_\mu^3 - g'B_\mu) \right]^2. \quad (6)$$

(6) can be rewritten as

$$-\frac{1}{2} M_Z^2 Z_\mu Z^\mu - \frac{1}{2} M_W^2 W_\mu^+ W^{\mu-},$$

provided we identify the gauge boson mass eigenstates and eigenvalues as

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2), \quad M_W = \frac{1}{2}gv. \\ Z_\mu &= (g^2 + g'^2)^{-1/2} (gW_\mu^3 - g'B_\mu), \quad M_Z = \frac{1}{2}(g^2 + g'^2)^{1/2}v. \\ A_\mu &= (g^2 + g'^2)^{-1/2} (g'W_\mu^3 + gB_\mu), \quad M_\gamma = 0. \end{aligned} \quad (7)$$

Let us introduce mixing parameters c_θ, s_θ , with $c_\theta^2 + s_\theta^2 = 1$, such that

$$c_\theta = g(g^2 + g'^2)^{-1/2}, \quad s_\theta = g'(g^2 + g'^2)^{-1/2},$$

i.e. $t_\theta \equiv s_\theta/c_\theta = g'/g$ and $Z_\mu = c_\theta W_\mu^3 - s_\theta B_\mu$ while $A_\mu = s_\theta W_\mu^3 + c_\theta B_\mu$. It follows that

$$\frac{M_W^2}{M_Z^2 c_\theta^2} = 1. \quad (8)$$

(8), in fact, turns out to be true (at the tree level) not only in the MSM but also in any $SU(2)_L \times U(1)_Y$ model with arbitrary elementary Higgs fields so long as only the $SU(2)_L$ doublets among them have neutral components acquiring VEVs. Indeed, it is valid even in condensate models without elementary scalar fields (e.g. technicolor [5]) provided there is a global custodial isospin [6] invariance protecting the symmetry-breaking sector. (8) is experimentally known to be quite accurate and we shall *always* assume it at the tree level. However, loop corrections make the LHS of (8) deviate from unity and then it is called the ρ -parameter, to be defined more precisely later.

(1) can now be rewritten, with f being a generic fermion field and $e = gs_\theta$, as

$$\begin{aligned} \mathcal{L}_{fg} &= i \sum_f \bar{f} \not{\partial} f + g/\sqrt{2} (J_{\mu L}^+ W^{\mu-} + h.c.) + \frac{g}{c_\theta} J_\mu^{NC} Z^\mu + e J_\mu^Q A^\mu \\ &= \sum_f i (f_L \not{\partial} f_L + \bar{f}_R \not{\partial} f_R) + \frac{e}{\sqrt{2} s_\theta} (J_{\mu L}^+ W^{\mu-} + h.c.) \\ &\quad + \frac{e}{s_\theta c_\theta} J_\mu^{NC} Z^\mu + e J_\mu^Q A^\mu. \end{aligned} \quad (9)$$

In (9) the weak charged, weak neutral and electromagnetic currents are respectively given, with $J_{\mu L}^- = (J_{\mu L}^+)^\dagger$, by

$$\begin{aligned} J_{\mu L}^+ = \sum_f \bar{f}_L \gamma_\mu T^+ f_L &= \sum_\ell \bar{\nu}_{eL} \gamma_\mu \ell_L + (\bar{u}_L \bar{c}_L \bar{t}_L) \gamma_\mu V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}, \\ &= \sum_\ell \bar{\nu}_L \frac{1}{2} \gamma_\mu (1 - \gamma_5) \ell + (\bar{u} \ \bar{c} \ \bar{t}) \frac{1}{2} \gamma_\mu (1 - \gamma_5) V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \end{aligned} \quad (10)$$

$$\begin{aligned}
J_\mu^{NC} = J_{\mu L}^3 - s_\theta^2 J_\mu^Q &= \sum_f (\bar{f}_L \gamma_\mu T_3 f_L - s_\theta^2 Q_f \bar{f} \gamma_\mu f) \\
&= \sum_f \bar{f} \frac{1}{2} \gamma_\mu [(T_3 - 2s_\theta^2 Q_f) - T_3 \gamma_5] f,
\end{aligned} \tag{11}$$

$$J_\mu^Q = \sum_f Q_f \bar{f} \gamma_\mu f = \sum_f Q_f (\bar{f}_L \gamma_\mu f_L + \bar{f}_R \gamma_\mu f_R). \tag{12}$$

In (10) ℓ sums over the charged leptons e, μ, τ while in (11) and (12) Q_f is the electromagnetic charge of f . $V_{CKM} = (V_{CKM}^\dagger)^{-1}$ is the unitary Cabibbo-Kobayashi-Maskawa 3×3 flavor matrix [7]:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

In the charged current $J_{\mu L}^\pm = J_{\mu L}^1 \pm i J_{\mu L}^2$ the quarks can change flavor through V_{CKM} while the neutral current is strictly flavor-conserving at the tree level by virtue of the GIM mechanism. On the other hand, the charged current is purely left-chiral whereas the neutral current has the chirally asymmetric combination $T_{3L} \gamma_\mu (1 - \gamma_5) - 2s_\theta^2 Q_f$. The latter is a vivid demonstration of the unification between the $(V - A)$ weak and the (Q) electromagnetic charges. We shall rewrite (12) as

$$J_\mu^{NC} = \frac{1}{2} \sum_f \bar{f} \gamma_\mu (v_f - a_f \gamma_5) f, \tag{13}$$

with

$$v_f = T_3 - 2Qs_\theta^2, \quad a_f = T_3.$$

Thus, for instance, $v_e = -\frac{1}{2}(1 - 4s_\theta^2)$ and $a_e = -1/2$.

At high energies (such as at LEP 1 with $\sqrt{s} \simeq 90$ GeV), the interactions among fermions and gauge bosons can be explicitly studied through distinct signatures of the W, Z bosons. At an energy or four momentum transfer sq. root $\sqrt{q^2}$ much below M_W, M_Z , however, only the effective four-fermion interaction, mediated by W, Z -exchange in a four-legged tree diagram (such as that for mu-decay, Fig. 3a or $\nu_\mu e$ elastic scattering, Fig. 3b) is available

for experimental study.

Fig. 3. Tree-level four-fermion weak processes

The effective interaction in the pointlike limit becomes

$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}} \left[J_{\mu L}^+ J_L^{\mu-} + J_{\mu}^{NC} J^{\mu NC} \right], \quad (14)$$

with the Fermi constant given by

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{4\pi\alpha_{EM}}{8s_\theta^2 c_\theta^2 M_Z^2}, \quad (15)$$

where α_{EM} is the fine structure constant. This means, of course, that $v = (\sqrt{2}G_F)^{-1/2} \simeq 246$ GeV. Low energy weak scattering and decay data are now known to conform to the MSM at an accuracy $\lesssim 5\%$.

Fermion mass terms arise as a consequence of the spontaneous symmetry breakdown from Yukawa interactions among f, \bar{f} and ϕ . They can be written as

$$- \sum_f m_f \bar{f} f.$$

The shift $\phi \rightarrow \phi' \equiv \phi - \langle \phi \rangle$ from unphysical fields to physical normal modes does two things. (1) It transforms the components ϕ^\pm and $\chi \equiv (\sqrt{2}i)^{-1}(\phi^0 - \bar{\phi}^0)$ into longitudinal (Goldstone) W_L^\pm and Z_L components respectively; (2) it leaves residually in the spectrum a single Higgs field $H = (\sqrt{2})^{-1}(\phi^0 + \bar{\phi}^0 - \sqrt{2}v)$ corresponding to a scalar particle of mass $m_H = (\sqrt{2}\lambda)^{1/2}G_F^{-1/2}$. The terms in the Lagrangian density, involving H , finally are

$$\begin{aligned} \frac{1}{2}(\partial_\mu H)^2 &- \frac{1}{2}m_H^2 H^2 - V_{SELF}(H) - (\sqrt{2}G_F)^{1/2} \sum_f m_f \bar{f} f \\ &+ \frac{G_F}{\sqrt{2}} (2M_W^2 W_\mu^+ W^{\mu-} + M_Z^2 Z^\mu Z_\mu) [2(G_F\sqrt{2})^{-1/2} H + H^2], \end{aligned}$$

$V_{SELF}(H)$ containing the self-interaction terms of H . Experimentally [8], from LEP 1, $m_H > 63.5$ GeV at present.

In the above we have discussed the electroweak MSM and its tree-level relations in terms of three free parameters g, g' and v . Equivalently, one may consider the measured constants α_{EM}, G_F and M_Z . Among these the fine structure constant is known most precisely from precision QED measurements:

$$\alpha_{EM}^{-1} = 137.0359895(61)$$

Next, G_F is known rather accurately from the charged current decay process (Fig. 4) controlling the μ -lifetime. In fact we will call it G_μ and use

Fig. 4. Muon decay diagrams

$$\begin{aligned} \frac{1}{\tau_\mu} = & \frac{G_\mu^2 m_\mu^5}{192\pi^3} (1 - 8m_e^2/m_\mu^2) \cdot \\ & \cdot \left[1 + \frac{3}{5} m_\mu^2/M_W^2 + (2\pi)^{-1} \alpha_{EM} \left\{ 1 + 2(3\pi)^{-1} \alpha_{EM} \ln(m_\mu/m_e) \right\} (25/4 - \pi^2) \right], \end{aligned}$$

to deduce

$$G_F = G_\mu = 1.166389(22) \times 10^{-5} \text{ (GeV)}^{-2}$$

from the observed value of τ_μ . There is also the crosssection σ for the neutral current induced elastic scattering process $\nu_\mu e \rightarrow \nu_\mu e$, with incident neutrino energy E_ν in the lab frame, enabling us to define a low-energy neutral current weak coupling:

$$G_{NC} \equiv \left[\frac{2\pi\sigma}{m_e E_\nu} \left(1 - 4s_\theta^2 + \frac{16}{3} s_\theta^4 \right)^{-1} \right]^{1/2} = \rho G_\mu. \quad (16)$$

(16) can also be taken as a precise experimental definition of the ρ -parameter which, as mentioned earlier, is unity at the tree level for models of our interest. It may also be noted that (15) can be recast as $\pi\alpha_{EM} = \sqrt{2}G_\mu M_W^2(1 - M_W^2/M_Z^2)$. On including radiative corrections, this generalizes to [9]

$$\sqrt{2}G_\mu M_W^2(1 - M_W^2/M_Z^2) = \pi\alpha_{EM}(1 - \Delta r)^{-1}, \quad (17)$$

where Δr is a radiative parameter.

Finally, LEP 1, with an integrated luminosity of $\sim 150 \text{ (pb)}^{-1}$, has given us a pretty accurate value of the Z -mass currently at a 7×10^{-5} precision level, namely

$$M_Z = 91.187(7) \text{ GeV}.$$

Of course, there are additional “dependent” parameters in the MSM which are directly known from experimental measurements, viz. the W -mass and s_θ^2 . The “best” values currently are

$$\begin{aligned} M_W &= 80.13(28) \text{ GeV}, \\ s_\theta^2 &= 0.2325(5). \end{aligned}$$

Among the fermion masses m_f , from chiral symmetry considerations, it is known that $5 \text{ MeV} < m_d < 15 \text{ MeV}$ and $2 \text{ MeV} < m_u < 8 \text{ MeV}$ with $0.25 < m_u/m_d < 0.70$. Moreover, similar considerations extended to K decays imply $100 \text{ MeV} \lesssim M_S \lesssim 300 \text{ MeV}$. Also, charmonium J/ψ and bottomonium Υ considerations imply $1.3 \text{ GeV} \leq m_c < 1.7 \text{ GeV}$, $4.7 \text{ GeV} < m_b < 5.3 \text{ GeV}$. The current limit on the top mass is $113 \text{ GeV} < m_t$. For leptons e - and μ -mass values are as appear in Ref. [4] while the τ -mass is now known to be $m_\tau = 1.777 \text{ GeV}$. The current upper limits on the neutrino masses are $m(\nu_e) < 7.2 \text{ eV}$, $m(\nu_\mu) < 250 \text{ keV}$ and $m(\nu_\tau) < 31 \text{ MeV}$.

• Basic LEP processes at the tree level [10]

Production and decay

We start by giving the basic gauge boson-fermion pair vertices. First note that, for a vanishing fermion mass, left- and right-chirality for a fermion corresponds to positive and negative helicity respectively. The second point to

remember is that gauge vertices (unlike Yukawa ones) do not change chirality since $\bar{f}\gamma_\mu f = \bar{f}_L\gamma_\mu f_L + \bar{f}_R\gamma_\mu f_R$ and $\bar{f}\gamma_\mu\gamma_5 f = -f_L\gamma_\mu f_L + \bar{f}_R\gamma_\mu f_R$. We will find it convenient to combine v_f and a_f of (13) into η_{Lf}, η_{Rf} :

$$\eta_{Lf} = v_f + a_f,$$

$$\eta_{Rf} = v_f - a_f.$$

Ignoring the CKM matrix V_{CKM} for quark flavor-mixing as a first approximation, we can take the basic gauge-boson-fermion-pair vertices as given in Fig. 5.

$$\begin{aligned} g(2c_\theta)^{-1}\bar{f}\gamma_\mu(v_f - a_f\gamma_5)f \\ = \frac{g}{2c_\theta}(\eta_{Lf}\bar{f}_L\gamma_\mu f_L + \eta_{Rf}\bar{f}_R\gamma_\mu f_R) \\ g(2\sqrt{2})^{-1}\bar{f}'\gamma_\mu(1 - \gamma_5)f = g(\sqrt{2})^{-1}\bar{f}'_L\gamma_\mu f_L \\ eQ_f\bar{f}\gamma_\mu f = eQ_f(\bar{f}_L\gamma_\mu f_L + \bar{f}_R\gamma_\mu f_R) \end{aligned}$$

Fig. 5. Basic gauge-boson-fermion-pair vertices.

Suppose we generically describe both W - and Z -vertices by $\hat{g}\bar{f}_2\gamma_\mu(v - a\gamma_5)f_1$. The tree level production cross section for the process

$$f_1(p_1)\bar{f}_2(p_2) \rightarrow V(p), \quad V = W, Z$$

with unpolarized beams and in the zero width approximation is given by

$$\sigma(f_1\bar{f}_2 \rightarrow V) = \overline{\sum}|T_{12}|^2\pi M_V^{-2}\delta(s - M_V^2), \quad (18)$$

with $s = (p_1 + p_2)^2$ as the square of the CM energy. As for the decay

$$V \rightarrow f_1\bar{f}_2,$$

the partial width is

$$\Gamma(V \rightarrow f_1\bar{f}_2) = \frac{|\vec{p}|}{8\pi}M_V^{-2}\overline{\sum}|T_{12}|^2, \quad (19)$$

where the CM fermion momentum $|\vec{p}| = \lambda(M_V^2, m_1^2, m_2^2) \equiv (2m_V)^{-1}(m_V^4 + m_1^4 + m_2^4 - 2m_V^2 M_V^2 - 2M_V^2 m_2^2 - 2m_1^2 m_2^2)^{\frac{1}{2}} \simeq M_V/2$ for $M_V \gg m_{1,2}$. In (18) and (19) $T_{1\bar{2}}$ is the transition matrix element $\hat{g}\bar{v}(p_2)\gamma^\mu(v - a\gamma_5)u(p_1)\epsilon_\mu^*$ in standard notation (our normalization is $\bar{u}u = \bar{v}v = 2m$) and \sum stands for the summation over all colors and spins while the bar on top implies division by initial spin and color factors. Thus

$$\begin{aligned} \overline{\sum}|T_{1\bar{2}}|^2 = \hat{g}^2 M_V^2 N_f^c \Big[(v^2 + a^2) \Big\{ 1 - \frac{1}{2} m_V^{-2} (m_1^2 + m_2^2) - \frac{1}{2} m_V^{-4} (m_1^2 - m_2^2)^2 \Big\} \\ + 3(v^2 - a^2) M_V^{-2} m_1 m_2 \Big]. \end{aligned}$$

The color factor N_f^c is 3 for quarks and 1 for leptons.

For light fermions, substituting the right value of \hat{g} for CC and NC interactions, we have

$$\Gamma(W \rightarrow f_1 \bar{f}_2) = \frac{\sqrt{2}}{12\pi} G_\mu M_W^3 N_f^C |V^{12}|^2, \quad (20a)$$

$$\Gamma(Z \rightarrow f \bar{f}) = \frac{\sqrt{2}}{12\pi} G_\mu M_Z^3 N_f^C (v_f^2 + a_f^2) = \frac{\alpha_{EM}}{6s_\theta^2 c_\theta^2} (T_{3f} - s_\theta^2 Q_f)^2 N_f^C, \quad (20b)$$

where V^{12} is unity when f is a lepton and equals V_{CKM}^{12} when f is a quark. Interestingly, in the limit $s_\theta \rightarrow 0$, $M_Z \rightarrow M_W$ and one has the sumrule $\Gamma(W \rightarrow f_1 \bar{f}_2) \rightarrow \Gamma(Z \rightarrow f_1 \bar{f}_1) + \Gamma(Z \rightarrow f_2 \bar{f}_2)$. Three remarks are in order. (1) Since the top is heavier than 113 GeV, the decays $W \rightarrow tb$, $Z \rightarrow t\bar{t}$ are not possible, (2) LEP 1 has excluded any extra heavy fermion upto a mass of ~ 45 GeV and (3) the heaviest known fermion, viz. the b -quark, weighs only 4.8 GeV, so that fermion mass terms in W - and Z -decay are mostly neglected. We can consider Z decays into particle-antiparticle pairs not only of charged leptons but also of neutrinos ν_e, ν_μ or ν_τ and define $\Gamma(Z \rightarrow \text{invisible}) = 3\Gamma(Z \rightarrow \nu\bar{\nu})$; similarly we can write $\Gamma(Z \rightarrow \text{hadrons}) = \sum_{5q} \Gamma(Z \rightarrow q\bar{q})$ where \sum_{5q} is over u, d, s, c and b . Finally,

$$\Gamma_Z^{tot} \simeq \sum_f \Gamma(Z \rightarrow f \bar{f}) = \sum_f \Gamma(Z \rightarrow \nu\bar{\nu}) (1 - 4|Q_f|s_\theta^2 + 8Q_f^4 s_\theta^4). \quad (21)$$

It may be noted that, m_H being > 63.5 GeV, any contribution to Γ_Z^{tot} from a final state containing H would be quite small. One can define $\Gamma(Z \rightarrow$

invisible) = $N_\nu \Gamma(Z \rightarrow \nu \bar{\nu})$ and experimentally determine N_ν from LEP 1. The current result is $N_\nu = 3 \pm 0.04$.

Let us move on to Z -production in e^+e^- collisions. We stick with the light fermion approximation. Thus, from (19), with the notation Γ_Z^f for $\Gamma(Z \rightarrow f\bar{f})$,

$$\sum |T_{12}|^2 = 48\pi M_V^2 M_Z^{-1} \Gamma_Z^f.$$

(18) now implies that – in the zero-width approximation –

$$\sigma(e^+e^- \rightarrow Z) = 12\pi^2 M_Z^{-1} \Gamma_Z^e \delta(s - M_Z^2),$$

a formula – which for finite widths – has to be modified via

$$\pi \delta(s - M_Z^2) \rightarrow M_Z \Gamma_Z^{tot} [(s - M_Z^2)^2 + M_Z^2 (\Gamma_Z^{tot})^2]^{-1}$$

to the Breit-Wigner form

$$\sigma(e^+e^- \rightarrow Z) = 12\pi \Gamma_Z^e \Gamma_Z^{tot} [(s - M_Z^2)^2 + M_Z^2 (\Gamma_Z^{tot})^2]^{-1}. \quad (22)$$

Near the Z -resonance,

$$\begin{aligned} \sigma(e^+e^- \rightarrow Z \rightarrow f\bar{f}) &= \sigma(e^+e^- \rightarrow Z) \Gamma_Z^f (\Gamma_Z^{tot})^{-1} \\ &= 12\pi \Gamma_Z^e \Gamma_Z^f [(s - M_Z^2)^2 + M_Z^2 (\Gamma_Z^{tot})^2]^{-1}. \end{aligned} \quad (23)$$

Scattering cross sections

Consider the process

$$e^-(p_1) e^+(p_2) \longrightarrow f(q_1) \bar{f}(q_2)$$

in the Born approximation involving γ^* and Z^* exchange in the s -channel, as shown in Fig. 6. Because of chirality-conservation at each vertex, there are four possible independent transition amplitudes: $T_{h_e} T_{h_f} = T_{LL}, T_{LR}, T_{RL}, T_{RR}$. Here h_e, h_f refer the handedness of the e, f being L or R . Thus we have

Fig. 6. Tree diagrams for $e^+e^- \rightarrow f\bar{f}$.

$$\frac{d\sigma}{d\cos\Theta} = \frac{s}{48\pi} N_f^C \sum_{spins} |T_{hehf}|^2, \quad (24)$$

with Θ as the CM scattering angle. Employing the η -notation introduced earlier,

$$|T_{hehf}|^2 = \frac{3}{8} (1 \pm \cos\Theta)^2 |\eta_{he}\eta_{hf} \sqrt{2} G_\mu M_Z^2 (s - M_Z^2 + iM_Z\Gamma_Z)^{-1} + 4\pi\alpha Q_e Q_f s^{-1}|^2,$$

with the $+$ ($-$) sign being for T_{LL} and T_{RR} (T_{LR} and T_{RL}).

(24) can now be rewritten as

$$\frac{d\sigma}{d\cos\Theta} = \frac{d\sigma^\gamma}{d\cos\Theta} + \frac{d\sigma^Z}{d\cos\Theta} + \frac{d\sigma^{\gamma Z}}{d\cos\Theta},$$

the individual pieces standing for the pure QED , the pure weak and the electroweak interference terms respectively. The first and the second pieces dominate at low energies and near the Z -resonance respectively. To show these pieces in detail, we define

$$\mathcal{R}(s) \equiv s[s - M_Z^2 + iM_Z\Gamma_Z^{tot}]^{-1}$$

and write

$$\begin{aligned} \frac{d\sigma^\gamma}{d\cos\Theta} &= \frac{\pi\alpha_{EM}^2}{2s} Q_f^2 N_f^C (1 + \cos^2\Theta), \\ \frac{d\sigma^Z}{d\cos\Theta} &= \frac{G_\mu^2 M_Z^4}{16\pi s} N_f^C |\mathcal{R}(s)|^2 [(v_e^2 + a_e^2)(v_f^2 + a_f^2)(1 + \cos^2\Theta) + 8a_e a_f v_e v_f \cos\Theta], \\ \frac{d\sigma^{\gamma Z}}{d\cos\Theta} &= -\frac{\alpha_{EM} \sqrt{2} G_\mu M_Z^2}{4s} Q_f N_f^C \text{Re } \mathcal{R}(s) [v_e v_f (1 + \cos^2\Theta) + 2a_e a_f \cos\Theta]. \end{aligned} \quad (25)$$

It is noteworthy that the terms linear in $\cos \Theta$ in the RHS of (25) would vanish if there was no axial vector coupling in weak interactions.

The total cross section splits into pure vector and axial vector pieces:

$$\begin{aligned}
\sigma_{f\bar{f}} &= \int_{-1}^1 d\cos \Theta \frac{d\sigma}{d\cos \Theta} = \sigma_{f\bar{f}}^{VV} + \sigma_{f\bar{f}}^{AA}, \\
\sigma_{f\bar{f}}^{VV} &= \frac{4\pi\alpha_{EM}^2 Q_f^2 N_f^C}{3s} - \frac{2\alpha_{EM}\sqrt{2}G_\mu M_Z^2}{3s} Q_f N_f^C \operatorname{Re} \mathcal{R}(s) v_e v_f \\
&\quad + \frac{G_\mu^2 M_Z^4}{6\pi s} N_f^C |\mathcal{R}|^2 (v_e^2 + a_e^2) v_f^2, \\
\sigma_{f\bar{f}}^{AA} &= \frac{G_\mu^2 M_Z^4}{6\pi s} N_f^C |\mathcal{R}|^2 (v_e^2 + a_e^2) a_f^2.
\end{aligned} \tag{26}$$

Near the Z -resonance the total cross section can be written as

$$\sigma_{f\bar{f}} \simeq \sigma_{f\bar{f}}^Z \left[1 + \frac{8\pi\alpha_{EM} Q_e Q_f}{\sqrt{2}G_\mu M_Z^2} \frac{v_e v_f}{(v_e^2 + a_e^2)(v_f^2 + a_f^2)} \frac{s - M_Z^2}{s} \right] + \sigma_{f\bar{f}}^\gamma. \tag{27}$$

On resonance, where $s = M_Z^2$, the background term

$$\sigma_{f\bar{f}}^\gamma = \frac{4\pi\alpha_{EM}^2 Q_f^2}{3M_Z^2} N_f^C$$

is a $< 1\%$ correction to the dominant term

$$\sigma_{f\bar{f}}^Z = \frac{3G_\mu^2 M_Z^4}{4\pi[\Gamma_Z^{tot}]^2} N_f^C (v_e^2 + a_e^2)(v_f^2 + a_f^2). \tag{28}$$

It should be pointed out that the detailed fitting of near-resonance Z line-shape requires higher order corrections. In particular, these oblige one to use an energy-dependent width $\Gamma_Z^{tot}(s)$.

Asymmetries

The differential cross section, discussed above, has a term that is even in $\cos \Theta$ and one that is odd in $\cos \Theta$

$$\begin{aligned}
\frac{d\sigma_{f\bar{f}}}{d\cos \Theta} &= \sigma_{f\bar{f}}(s) \frac{3}{8} (1 + \cos^2 \Theta) + \cos \Theta \left[\frac{G_\mu^2 M_Z^4}{2\pi s} N_f^C |\mathcal{R}|^2 a_e a_f v_e v_f \right. \\
&\quad \left. - \frac{\alpha_{EM}\sqrt{2}G_\mu M_Z^2}{2s} Q_f N_f^C (\operatorname{Re} \mathcal{R}) a_e a_f \right] \\
&\equiv \frac{3}{8} \sigma_{f\bar{f}}(s) (1 + \cos^2 \Theta) + \Delta_{f\bar{f}}^0(s) \cos \Theta.
\end{aligned} \tag{29}$$

Because of the odd term, a forward-backward asymmetry, viz.

$$\mathcal{A}_{f\bar{f}}^{FB}(s, \cos \Theta) = \frac{d\sigma_{f\bar{f}}(\Theta) - d\sigma_{f\bar{f}}(\pi - \Theta)}{d\sigma_{f\bar{f}}(\Theta) + d\sigma_{f\bar{f}}(\pi - \Theta)} = \frac{8}{3} \frac{\Delta_{f\bar{f}}(s)}{\sigma_{f\bar{f}}(s)} \frac{\cos \Theta}{1 + \cos^2 \Theta} \quad (30)$$

is generated. There is an angular-integrated version of the forward-backward asymmetry, namely

$$A_{f\bar{f}}^{FB}(s) = \frac{(\int_0^1 - \int_{-1}^0) d\cos \Theta \frac{d\sigma_{f\bar{f}}}{d\cos \Theta}}{\int_{-1}^1 d\cos \Theta \frac{d\sigma_{f\bar{f}}}{d\cos \Theta}} = \frac{\Delta_{f\bar{f}}(s)}{\sigma_{f\bar{f}}(s)}. \quad (31)$$

This is, of course, easier to measure because of the higher statistics. The general tree-level expressions for the forward-backward asymmetry appear in eqs. (29)-(31), but it is useful to consider two special energy domains:

(1) Small $s \ll M_Z^2$.

Now

$$\sigma_{f\bar{f}} \simeq \frac{4\pi\alpha_{EM}^2}{3s} \left[Q_f^2 N_f^C + \frac{\sqrt{2}G_\mu}{2\pi\alpha_{EM}} N_f^C Q_f \frac{v_e v_f}{1 - sM_Z^{-2}} \right]. \quad (32)$$

In (32) the factor outside is the QED “point cross section” used to normalize σ ($e^+e^- \rightarrow \text{hadrons}$). Furthermore,

$$A_{f\bar{f}}^{FB}(s) \simeq \frac{3}{8} \frac{a_e a_f}{Q_f} \frac{\sqrt{2}G_\mu}{\pi\alpha_{EM}} \frac{s}{1 - sM_Z^{-2}} \quad (33)$$

which vanishes as $s \rightarrow 0$.

(2) For $s \simeq M_Z^2$ we find

$$A_{f\bar{f}}^{FB}(M_Z^2) = \frac{\Delta_{f\bar{f}}^0(M_Z^2)}{\sigma_{f\bar{f}}(M_Z^2)} = \frac{3}{4} \frac{2v_e a_e}{v_e^2 + a_e^2} \frac{2v_f a_f}{v_f^2 + a_f^2} = \frac{3}{4} \frac{\eta_{Le}^2 - \eta_{Re}^2}{\eta_{Le}^2 + \eta_{Re}^2} \frac{\eta_{Lf}^2 - \eta_{Rf}^2}{\eta_{Lf}^2 + \eta_{Rf}^2}. \quad (34)$$

It is important to note that

$$\frac{\eta_{L\ell}^2 - \eta_{R\ell}^2}{\eta_{L\ell}^2 + \eta_{R\ell}^2} = \frac{2\xi}{1 + \xi^2},$$

with $\xi = 1 - 4s_\theta^2 \simeq 0.10$ for $\ell = e, \mu, \tau$. Thus

$$A_{\mu\bar{\mu}}^{FB}(M_Z^2) = \frac{3\xi^2}{(1 + \xi^2)^2}. \quad (35)$$

Near $s = M_Z^2$,

$$A_{ff}^{FB}(s) \simeq \frac{3v_e a_e v_f a_f}{(v_e^2 + a_e^2)(v_f^2 + a_f^2)} - \frac{3}{2\pi} \frac{\sqrt{2} G_\mu M_Z^2}{\alpha_{EM}} (1 - M_Z^2 s^{-1}) \frac{a_e a_f}{(v_e^2 + a_e^2)(v_f^2 + a_f^2)}. \quad (35a)$$

Another interesting asymmetry concerns the polarization of the final state fermion f (such as a τ):

$$A_{pol}^f \equiv \frac{\sigma(e^+ e^- \rightarrow f_L \bar{f}) - \sigma(e^+ e^- \rightarrow f_R \bar{f})}{\sigma(e^+ e^- \rightarrow f_L \bar{f}) + \sigma(e^+ e^- \rightarrow f_R \bar{f})}.$$

In fact, on the Z , this yields

$$A_{pol}^f(M_Z^2) = \frac{\eta_{Lf}^2 - \eta_{Rf}^2}{\eta_{Lf}^2 + \eta_{Rf}^2},$$

independently of initial couplings. It may be noted that $A_{FB}^f(M_Z^2) = \frac{3}{4} A_{LR}^e(M_Z^2) A_{LR}^f(M_Z^2)$. The experimental measurement of A_{pol}^f is most feasible for $f = \tau$ since the τ -polarization can be measured from the decays $\tau \rightarrow \pi\nu, \rho\nu, a_1\nu$ with the subsequent decays $\rho \rightarrow \pi\pi$, $a_1 \rightarrow 3\pi$ as well as from $\tau \rightarrow \nu + \text{jets}$. For $e^+ e^- \rightarrow \tau^+ \tau^-$,

$$A_{pol}^\tau(M_Z^2) = \frac{2\xi}{1 + \xi^2}.$$

There are additional asymmetries involving polarized beams which are of great theoretical interest and await futuristic experiments. But since clean polarized high energy e^\pm beams will not be available for some time, we do not discuss them in detail except to define

$$A_{LR} \equiv \frac{\sigma(e_L^- e^+ \rightarrow f \bar{f}) - \sigma(e_R^- e^+ \rightarrow f \bar{f})}{\sigma(e_L^- e^+ \rightarrow f \bar{f}) + \sigma(e_R^- e^+ \rightarrow f \bar{f})}$$

and quote

$$A_{LR}(M_Z^2) = \frac{\eta_{Le}^2 - \eta_{Re}^2}{\eta_{Le}^2 + \eta_{Re}^2} = \frac{2\xi}{1 + \xi^2}.$$

- **1-loop radiative corrections in the on-shell renormalization scheme**

What are our independent couplings? First, the fine structure constant α_{EM} and the mass M_Z . These define an on-shell QED-like [11] renormalization scheme [12]. Then there is the μ -decay coupling constant given at the tree level by

$$\sqrt{2}G_\mu = \frac{1}{v^2} = \frac{\pi\alpha_{EM}}{M_W^2 s_\theta^2}, \quad s_\theta^2 = 1 - M_W^2 M_Z^{-2}.$$

Note that the gauge couplings $g = \sqrt{4\pi\alpha_{EM}}/s_\theta$, $g' = \sqrt{4\pi\alpha_{EM}}/c_\theta$ are in the category of dependent parameters. Next, one has to renormalize these parameters. Finally come the field or wave function renormalizations.

The input parameters in the true bare Lagrangian are M_{Wb}^2 , M_{Zb}^2 , α_b and $G_{\mu b}$ where the subscript b signifies bare values. After the 1-loop correction, they become cutoff dependent (i.e. infinite). Then they have to be reparametrized in terms of the corresponding finite physical parameters by additive infinite renormalization constants. Thus,

$$\begin{aligned} M_{W,Zb}^2 &= M_{W,Z}^2 \left(1 + \frac{\delta M_{W,Z}^2}{M_{W,Z}^2} \right), \\ \alpha_{EMb} &= \alpha_{EM} \left(1 + \frac{\delta \alpha_{EM}}{\alpha_{EM}} \right), \\ s_{\theta b}^2 &= s_\theta^2 \left(1 + \frac{\delta s_\theta^2}{s_\theta^2} \right), \\ G_{\mu b} &= G_\mu \left(1 + \frac{\delta G_\mu}{G_\mu} \right). \end{aligned} \tag{36}$$

Finally, we have

$$\frac{\delta G_\mu}{G_\mu} = \frac{\delta \alpha_{EM}}{\alpha_{EM}} - \frac{\delta M_W^2}{M_W^2} - \frac{\delta s_\theta^2}{s_\theta^2}. \tag{37}$$

Turning to field renormalization, we ignore the infrared problem due to soft photons in QED. That is an old subject and is understood in terms of standard textbook techniques. We simply attribute a mass m_γ to the photon as an infrared regulator and require that the limit $m_\gamma \rightarrow 0$ must exist for all observable quantities.

For fields, then, write the renormalized objects as

$$\begin{aligned} V_{\mu b} &= \sqrt{Z_V} V_{\mu r} \quad (V = A, W^\pm, Z), \\ f_b &= \sqrt{Z_f} f_r, \\ H_b &= \sqrt{Z_H} H_r, \end{aligned} \tag{38}$$

where the wave function renormalization constants Z are fixed by the condition that *propagators of the renormalized fields have unit residues at their poles*. To leading order, $Z_i = 1$ and we may write

$$Z_i = 1 + \delta Z_i; \quad \sqrt{Z_i} \simeq 1 + \frac{1}{2} \delta Z_i + \dots$$

The actual renormalization procedure of a physical amplitude can be done as follows. First, perform the parameter shifts and field renormalizations to 1-loop by expanding upto linear order. Thus, for the $f\bar{f}V$ vertices substitute

| | |
|--|---|
| (bare) | (renormalized) |
| $eQ_f\gamma_\mu$ | $\rightarrow eQ_f\gamma_\mu \left(1 + \frac{1}{2}\delta Z_A + \delta Z_f + \frac{\delta e}{e}\right)$ |
| $\sqrt{2G_\mu}M_Z\gamma^\mu[T_{3f}(1 - \gamma_5) - 2s_\theta^2 Q_f]$ | $\rightarrow \sqrt{2G_\mu}M_Z\gamma^\mu[T_3(1 - \gamma_5)$ $- 2Q_f s_\theta^2(1 + \delta s_\theta^2/s_\theta^2)] \cdot$ $\left(1 + \frac{1}{2}\delta Z_Z + \delta Z_f + \frac{1}{2}\frac{\delta M_Z^2}{M_Z^2} + \frac{1}{2}\frac{\delta G_\mu}{G_\mu}\right)$ |
| $\sqrt{2\sqrt{2}G_\mu}M_W\gamma^\mu(1 - \gamma_5)$ | $\rightarrow \sqrt{2\sqrt{2}G_\mu}M_W\gamma^\mu(1 - \gamma_5) \left(1 + \frac{1}{2}\delta Z_W\right.$ $\left. + \frac{1}{2}\delta Z_{f1} + \frac{1}{2}\delta Z_{f2} + \frac{1}{2}\frac{\delta M_W^2}{M_W^2} + \frac{1}{2}\frac{\delta G_\mu}{G_\mu}\right).$ |

Analogous substitutions have to be made for other vertices.

Next, the mass counter terms and the wave function factors have to be introduced. These are determined by the physical tranverse parts of the vector boson self-energy:

$$D_{VV}^{\mu\nu}(q^2) = \frac{-i\eta^{\mu\nu}}{q^2 - M_V^2 - \Pi_{VV}(q^2)} + q^\mu q^\nu \text{ term.} \tag{39}$$

In (39) the $\eta^{\mu\nu}$ -term signifies the tranverse part whereas the unspecified $q^\mu q^\nu$ term stands for the longitudinal part. $D_{VV}^{\mu\nu}$ arises from a sum of

Fig. 7. Perturbation series for vector boson self-energies.

the bare propagator and vacuum polarization bubbles as shown in Fig. 7. Here Π_{VV} is defined as the coefficient of $(-i\eta^{\mu\nu})$ in the 1-loop vector boson self-energies:

$$\Pi^{\mu\nu}(q) = i\Pi_{VV}(q^2)\eta^{\mu\nu} + q^\mu q^\nu \text{ term.}$$

Thus

$$\begin{aligned} \frac{-i\eta^{\mu\nu}}{q^2 - M_V^2} &= \frac{-i\eta^{\mu\nu}}{q^2 - M_V^2 - \Pi_{VV}(q^2)} \cdot \\ &\left[1 + \Pi_{VV}(q^2) \frac{1}{q^2 - M_V^2} + \Pi_{VV}(q^2) \frac{1}{q^2 - M_V^2} \Pi_{VV} \frac{1}{q^2 - M_V^2} + \cdots \right]. \end{aligned}$$

Since the vector boson self-energy is quadratically divergent, two subtractions (chosen on-shell) are needed so that

$$\Pi_{VVr}(q^2) = \Pi_{VV}(q^2) - \Pi_{VV}(M_V^2) - (q^2 - M_V^2) \frac{d\Pi_{VV}(M_V^2)}{dq^2} + \text{higher orders.}$$

(Note that the unrenormalized Π -function is a divergent, regulator-dependent quantity). The tranverse part of the free inverse propagator is changed as under

$$\begin{aligned} i\eta^{\mu\nu}(q^2 - M_{Vb}^2) &\rightarrow i\eta^{\mu\nu} Z_V(q^2 - M_V^2 - \delta M_V^2) \\ &= i\eta^{\mu\nu} [q^2 - M_V^2 - \delta M_V^2 + \delta Z_V(q^2 - M_V^2) + \cdots], \end{aligned}$$

where $\delta Z_V = Z_V - 1$. Diagrammatically, this entry of the counter terms is shown in Fig. 8.

Fig. 8. Composition of renormalized vector boson propagator.

On mass-shell renormalization now implies that the transverse part of the vector boson renormalized self-energy and its derivative vanish at $q^2 = M_V^2$. This means

$$\begin{aligned}\delta M_V^2 &= -\text{Re } \Pi_{VV}(M_V^2) \\ \delta Z_V \equiv Z_V - 1 &= +\text{Re } \frac{d\Pi_{VV}}{dq}(M_V^2).\end{aligned}\tag{40}$$

Since Z, W are unstable, the self-energy has an imaginary part too: $\text{Im } \Pi_{VV}(M_V^2) \equiv M_V \Gamma_V$ giving the total width of V . We will henceforth drop the prefix Re.

The above treatment, though exactly valid for W , has to be modified both for the photon and the Z because of $\gamma - Z$ mixing. In place of (39), one would now have a 2×2 $\gamma - Z$ symmetric inverse propagator matrix [13]

$$\hat{D}^{-1} = \begin{pmatrix} q^2 - \Pi_{\gamma\gamma}(q^2) & -\Pi_{\gamma Z}(q^2) \\ -\Pi_{\gamma Z}(q^2) & q^2 - M_Z^2 - \Pi_{ZZ}(q^2) \end{pmatrix}\tag{41}$$

Taking inverse and keeping only linear terms in Π (consistent to 1-loop), we have

$$\begin{aligned}D_{\gamma\gamma} &\simeq \frac{1}{q^2} \left[1 + \frac{\Pi_{\gamma\gamma}(q^2)}{q^2} \right], \\ D_{\gamma Z} &\simeq \frac{\Pi_{\gamma Z}(q^2)}{q^2(q^2 - M_Z^2)}, \\ D_{ZZ} &\simeq \frac{1}{q^2 - M_Z^2} \left[1 + \frac{\Pi_{ZZ}(q^2)}{q^2 - M_Z^2} \right].\end{aligned}\tag{42}$$

(We shall have more to say about these pi-functions later on.) All the vector boson self-energy diagrams in a general gauge are given in Fig. 9.

Fig. 9. Vector boson self-energies in a general gauge.

Of course, the tadpole contributions drop out of renormalized quantities.

Finally, we have to come to the fermions. The fermion propagator renormalization is straightforward (Fig. 10) except that it has to be separate

Fig. 10. Fermion self-energy diagrams.

for the different chiral components. Equivalently, we can write

$$\delta Z_f = \delta Z_{vf} + \delta Z_{af} \gamma_5. \quad (43)$$

Coming to the fermionic electric charge (Fig. 11), the condition is that, to

Fig. 11. Fermion electric charge renormalization.

1-loop, the EM coefficient of the γ_μ vertex must lead in the zero photon energy limit ($q^2 = 0$) to the renormalized charge e . The vertex correction diagrams to 1-loop are shown in Fig. 12.

Fig. 12. 1-loop vertex corrections to fermion electric charge.

The renormalization condition, together with the EM Ward-identity following from current-conservation $\partial_\mu J^{\mu EM} = 0$, leads to a constraint linking some vertex correction contributions to self-energy ones, but we shall not go into these details. They may be found in the review article by Jegerlehner [12].

We want to end this discussion with remarks on the radiative corrections to G_μ , measured in the decay $\mu \rightarrow e \bar{\nu}_e \nu_\mu$. The effective pointlike interaction is

$$-4 \frac{G_\mu}{\sqrt{2}} \bar{u}_{\nu_\mu} \gamma_\alpha \frac{1}{2} (1 - \gamma_5) u_\mu \bar{u}_e \gamma^\alpha \frac{1}{2} (1 - \gamma_5) v_{\nu_e}.$$

The 1-loop radiative corrections come from vector boson self-energy diagrams, vertex corrections and box diagrams. These are schematically enumerated along with the tree graph in Fig. 13. In more specific detail,

Fig. 13. Schematic enumeration μ -decay diagrams to 1-loop.

the vertex corrections (top line) and the box contributions are shown in Fig. 14.

Fig. 14. Vertex corrections and box contributions to μ -decay at 1-loop.

To one loop, one can approximate $[M_W^2 + \Pi_{WWr}(0)]^{-1}$ to $M_W^{-2}[1 - M_W^{-2}\Pi_{WWr}(0)]$. Thus Fig. 13, rewritten as an equation, reads

$$\frac{G_\mu}{\sqrt{2}} = \frac{e_b^2}{8s_{\theta b}^2 M_{Wb}^2} \left\{ 1 - \frac{\Pi_{WWr}(0)}{M_W^2} + \delta_{VERTEX} + \delta_{BOX} \right\}. \quad (44)$$

Detailed expressions for δ_{VERTEX} and δ_{BOX} may be found in Refs. [12] and [14]. Rewriting all bare quantities in terms of the corresponding renormalized quantities and employing the trick $\delta c_\theta/s_\theta^2 = c_\theta^2 s_\theta^{-2} \delta c_\theta^2/c_\theta^2$, we can write

$$\begin{aligned} \frac{G_\mu}{\sqrt{2}} &= \frac{e^2}{8s_\theta^2 M_W^2} \left\{ 1 + \frac{\delta\alpha_{EM}}{\alpha_{EM}} - \frac{c_\theta^2}{s_\theta^2} \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right) - \frac{\delta M_W^2}{M_W^2} \right. \\ &\quad \left. - \frac{\Pi_{WWr}(0)}{M_W^2} + \delta_{VERTEX} + \delta_{BOX} \right\}, \\ &= \frac{\pi\alpha_{EM}}{2s_\theta^2 M_W^2} (1 + \Delta r), \end{aligned} \quad (45)$$

where $\Delta r = (\Delta r)_{SE} + (\Delta r)_{VERTEX} + (\Delta r)_{BOX}$.

We will not go into the detailed technical calculation of Δr for which the best reference is Hollick's article [12]. Suffice it to say that the vertex corections and box graph terms are an order of magnitude smaller and that the main contribution to Δr comes from the various self-energy sources. This

is a generic property of all EW radiative corrections of interest except the $Zb\bar{b}$ vertex. These vector boson self-energy corrections, which form a gauge-invariant subset, have been called [16] *oblique*. As an example let us look (Fig. 15) at the vacuum polarization contribution to the photon self-

Fig. 15. Fermion-summed photon vacuum polarization.

energy, summed over all fermions. This leads to

$$\frac{\Delta\alpha_{EM}}{\alpha_{EM}} = \frac{1}{3\pi} \sum_f Q_f^2 N_f^C \left(\ln \frac{M_Z^2}{m_f^2} - \frac{5}{3} \right), \quad (46)$$

where \sum_f covers both quarks and leptons. The RHS of (46) has a leptonic contribution which is straightforward. The nontop hadronic contribution should not be perturbatively calculated because of nonperturbative QCD effects and is best given by the dispersion integral

$$-\frac{M_Z^2}{3\pi} \operatorname{Re} \int_{4m_\pi^2}^{\infty} ds (s - M_Z^2 + i\epsilon)^{-1} [\sigma_{e^+e^- \rightarrow \mu^+\mu^-}(s)]^{-1} [\sigma_{e^+e^- \rightarrow \text{hadrons}}(s)].$$

On the other hand, the top contribution, owing to the high value of m_{top} , is perturbative. However, there is another source of the top contribution, namely the W self-energy (Fig. 16).

Fig. 16. Top contribution to W self-energy.

All told, the top-contribution to Δr is:

$$\Delta r^{top} = -\frac{\sqrt{2}G_\mu M_W^2}{16\pi^2} \left\{ 3\frac{c_\theta^2}{s_\theta^2} \frac{m_t^2}{M_W^2} + 2\left(\frac{c_\theta^2}{s_\theta^2} - \frac{1}{3}\right) \ell n \frac{m_t^2}{M_W^2} + \frac{4}{3} \ell n c_\theta^2 + \frac{c_\theta^2}{s_\theta^2} - \frac{7}{9} \right\} \quad (47)$$

and has a piece that is quadratic in m_t . The Higgs contribution, coming basically from the diagrams of Fig. 17, is

$$\Delta r^{Higgs} = \frac{\sqrt{2}G_\mu M_W^2}{16\pi^2} \cdot 113 \left(\ell n \frac{m_H^2}{M_W^2} - \frac{5}{6} \right).$$

Fig. 17. Higgs contributions to vector boson self-energies.

• STAR SCHEME AND OBLIQUE CORRECTIONS

Vacuum Polarizations

Though the on-mass-shell renormalization scheme is axiomatically and logically the clearest, there is an inconvenience. It is not well suited to take into account the running of coupling strengths as functions of q^2 in consequence of renormalization group evolution. This deficiency is removed by the star (\star) scheme of Kennedy and Lynn [17] which we adopt for the rest of the lectures. This preserves all the good features of the on-shell scheme. Yet, as will be clear below, it is able to write q^2 -dependent 1-loop physical amplitudes in terms of running coupling strengths.

First, we return to the pi-functions and define them for currents rather than vector bosons. We work in the $SU(2) \times U(1)$ theory and assume weak isospin invariance. Thus $\Pi^{AB}(q^2)$ is defined by

$$i\Pi_{\mu\nu}^{AB}(q) \equiv \int d^4x e^{iq \cdot x} \langle \Omega | J_\mu^A(x) J_\nu^B(0) | \Omega \rangle = i\Pi_{AB}(q^2) \eta_{\mu\nu} + q_\mu q_\nu \text{ terms.} \quad (48)$$

with (43) and (10) as well as the following figures, we have

$$\Pi_{\gamma\gamma} = e^2 \Pi_{QQ}, \quad (49a)$$

$$\Pi_{ZZ} = \frac{e^2}{c_\theta^2 s_\theta^2} (\Pi_{33} - 2s_\theta^2 \Pi_{3Q} + s_\theta^4 \Pi_{QQ}), \quad (49b)$$

$$\Pi_{WW} = \frac{e^2}{2s_\theta^2} (\Pi_{11} + \Pi_{22}) = \frac{e^2}{s_\theta^2} \Pi_{11}, \quad (49c)$$

$$\Pi_{\gamma Z} = \frac{e^2}{s_\theta c_\theta} (\Pi_{3Q} - s_\theta^2 \Pi_{QQ}). \quad (49d)$$

These are the unrenormalized Π -functions which are ultraviolet divergent and hence regulator-dependent. EM gauge invariance dictates that an on-shell photon is a pure state, i.e. $\Pi_{XQ} = 0 \forall X$.

Next, concentrate on $\Pi_{QQ}(q^2)$. Since the photon self-energy has no zero mass pole, $\Pi_{QQ}(q^2) \propto q^2$ as $q^2 \rightarrow 0$ and it is convenient to define

$$\Pi'_{QQ}(q^2) \equiv \frac{\Pi_{QQ}(q^2)}{q^2},$$

where $\Pi'_{QQ}(0)$ is finite. Now, from (39), the transverse part of the complete (dressed) photon propagator acting between two physical lines is

$$e^2 (D_{\gamma\gamma}^{\mu\nu})_{tr} = -\frac{i\eta^{\mu\nu}}{q^2(1 - e^2 \Pi'_{QQ}(q^2))} \equiv -\frac{i\eta^{\mu\nu}}{q^2} e_\star^2(q^2). \quad (50)$$

Fig. 18. Dressed photon propagator.

(50) introduces the running QED electric charge (or starred charge)

$$e_\star^2(q^2) \equiv \frac{e^2}{1 - e^2 \Pi'_{QQ}(q^2)} \simeq e^2[1 + e^2 \Pi'_{QQ}(q^2)], \quad (51)$$

the second step keeping only linear Π terms to 1-loop. The first RHS, by the way, is the classic formula of Gell-Mann and Low. Thus the value of α_{EM} , measured from $(g - 2)_e$ or the A.C. Josephson effect is $\alpha_{EM} \simeq \frac{1}{4\pi} e_\star^2(0)$ whereas that measured at LEP 1 on the mass of the Z corresponds to $\frac{1}{4\pi} e_\star^2(M_Z^2)$. In fact,

$$\alpha_{\star EM}^{-1}(q^2) = \alpha_{EM}^{-1} - 4\pi[\Pi'_{QQ}(q^2) - \Pi'_{QQ}(0)]. \quad (52)$$

The interesting point in (52) is that no explicit renormalization of the pi-function is necessary since the particular combination is UV finite. As a specific application of (52), consider the 1-loop fermionic contribution to $\alpha_\star(M_Z^2)$. An explicit calculation yields

$$\alpha_{\star EM}^{-1}(M_Z^2) - \alpha_{EM}^{-1} = -\frac{1}{3\pi} \sum_f Q_f^2 N_f^C \left(\ell n \frac{M_Z^2}{M_f^2} - \frac{5}{3} \right). \quad (53)$$

It is noteworthy that the RHS of (53) is the same as $\delta\alpha_{EM}/\alpha_{EM}^2$ in the on-shell renormalization scheme. The evolution in (53), with all known fermions put in, brings α_{EM}^{-1} from $\simeq 137.0$ down to $\alpha_{\star EM}^{-1}(M_Z^2) \simeq 128.8$ and this is in brilliant agreement with experiment.

Scattering

Next, let us do a similar exercise for the neutral current coupling. We have done a detailed analysis for the CC induced muon decay in the on-shell scheme. That is convenient there since all relevant energies are low. In contrast, physical neutral current scattering (like CC scattering) is a highly q^2 -dependent phenomenon and a treatment of its renormalization in the star scheme is instructive. The matrix element of the NC interaction between

two physical scattering states can be formally written to 1-loop as in Fig. 19.

Fig. 19. NC scattering with 1-loop oblique corrections.

We exclude Lorentz indices and spinor structures since we want to obtain a compact form for the vacuum polarization insertions which only modify the gauge boson propagators. This is why they are called “oblique” as opposed to the “direct” vertex and box graph corrections. It is also a fact that, except in the $Zb\bar{b}$ vertex, these are the only significant 1-loop contributions in LEP physics. Let us use bare masses $m_{\gamma b}^2 = 0$, $m_{Wb}^2 = e^2 v^2 / (4s_\theta^2)$, $m_{Zb}^2 = e^2 v^2 / (4s_\theta^2 c_\theta^2)$. Thus we can write (including the QED part):

$$\begin{aligned}
M_{NC} = & e^2 Q \frac{-i}{q^2} Q' + e^2 Q \frac{-i}{q^2} i \Pi_{\gamma\gamma}(q^2) \frac{-i}{q^2} Q' \\
& + \frac{e^2}{c_\theta^2 s_\theta^2} (T_3 - s_\theta^2 Q) \frac{-i}{q^2 - M_{Zb}^2} (T'_3 - s_\theta^2 Q') \\
& + \frac{e^2}{c_\theta^2 s_\theta^2} (T_3 - Q s_\theta^2) \frac{-i}{q^2 - M_{Zb}^2} \Pi_{ZZ}(q^2) \frac{-i}{q^2 - M_{Zb}^2} (T'_3 - Q' s_\theta^2) \\
& + \frac{e^2}{c_\theta s_\theta} Q \frac{-i}{q^2} i \Pi_{\gamma Z}(q^2) \frac{-i}{q^2 - M_{Zb}^2} (T'_3 - Q' s_\theta^2) \\
& + \frac{e^2}{c_\theta s_\theta} (T_3 - Q s_\theta^2) \frac{-i}{q^2 - M_{Zb}^2} i \Pi_{\gamma Z}(q^2) \frac{-i}{q^2} Q'
\end{aligned}$$

$$\begin{aligned}
&= e^2 Q Q' \left(\frac{-i}{q^2} \right) \left[1 + \frac{\Pi_{\gamma\gamma}(q^2)}{q^2} \right] + \frac{e^2}{s_\theta^2 c_\theta^2} (T_3 - s_\theta^2 Q) \cdot \\
&\quad (T'_3 - s_\theta^2 Q') \left(\frac{-i}{q^2 - M_{Zb}^2} \right) \left[1 + \frac{\Pi_{ZZ}(q^2)}{q^2 - M_{Zb}^2} \right] \\
&\quad + \frac{e^2}{c_\theta s_\theta} \left[Q(T'_3 - Q' s_\theta^2) + Q'(T_3 - Q s_\theta^2) \right] \left(\frac{-i}{q^2 - M_{Zb}^2} \right) \Pi'_{\gamma Z}(q^2) \\
&\simeq e_\star^2(q^2) Q \left(\frac{-i}{q^2} \right) Q' + \frac{e^2}{s_\theta^2 c_\theta^2} (T_3 - s_\theta^2 Q) \frac{-i}{q^2 - M_{Zb}^2 - \Pi_{ZZ}(q^2)} \cdot \\
&\quad \left[T_3 - Q' s_\theta^2 \left(1 - \frac{c_\theta}{s_\theta} \Pi'_{\gamma Z}(q^2) \right) \right] \\
&\quad + \frac{e^2}{s_\theta^2 c_\theta^2} \left[T_3 - Q s_\theta^2 \left(1 - \frac{c_\theta}{s_\theta} \Pi'_{\gamma Z}(q^2) \right) \right] (T'_3 - s_\theta^2 Q').
\end{aligned}$$

Now define $M^2(q^2) \equiv M_{Zb}^2 + \Pi_{ZZ}(q^2)$. At $q^2 = M_Z^2$ this becomes the on-mass shell renormalization scheme definition $M_Z^2 = M_{Zb}^2 + \Pi_{ZZ}(M_Z^2)$. Thus

$$M^2(q^2) = M_Z^2 + \Pi_{ZZ}(q^2) - \Pi_{ZZ}(M_Z^2). \quad (54)$$

Also define

$$s_\star^2(q^2) \equiv s_\theta^2 \left(1 - \frac{c_\theta}{s_\theta} \Pi'_{\gamma Z}(q^2) \right), \quad (55)$$

i.e.

$$c_\star^2(q^2) \equiv c_\theta^2 \left(1 - \frac{s_\theta}{c_\theta} \Pi'_{\gamma Z}(q^2) \right).$$

Since we are working to linear terms in Π ,

$$\begin{aligned}
&(T_3 - s_\theta^2 Q) \left[T'_3 - Q' s_\theta^2 \left(1 - \frac{c_\theta}{s_\theta} \Pi'_{\gamma Z}(q^2) \right) \right] \\
&\quad + \left[T_3 - Q s_\theta^2 \left(1 - \frac{c_\theta}{s_\theta} \Pi'_{\gamma Z}(q^2) \right) \right] (T'_3 - Q' s_\theta^2) \\
&\simeq \left[T_3 - Q s_\theta^2 \left(1 - \frac{c_\theta}{s_\theta} \Pi'_{\gamma Z}(q^2) \right) \right] \left[T'_3 - Q' s_\theta^2 \left(1 - \frac{c_\theta}{s_\theta} \Pi'_{\gamma Z}(q^2) \right) \right] \\
&= (T_3 - Q s_\star^2(q^2)) (T'_3 - Q' s_\star^2(q^2)).
\end{aligned}$$

One can then rewrite M_{NC} as follows:

$$M_{NC} = e_\star^2(q^2)Q \left(-\frac{1}{q^2} \right) Q' + \frac{e^2}{c_\theta^2 s_\theta^2} [T_3 - Q s_\star^2(q^2)] \frac{-i}{q^2 - M^2(q^2)} [T'_3 - Q' s_\star^2(q^2)].$$

We define the Z wave function renormalization constant as the residue of the pole at $q^2 = M_Z^2$ of i times the Z -propagator, i.e.

$$\frac{1}{q^2 - M_Z^2 - \Pi_{ZZ}(q^2) + \Pi_{ZZ}(M_Z^2)} \equiv \frac{Z_Z}{q^2 - M_{Z^\star}^2(q^2)},$$

where

$$Z_Z \simeq 1 + \frac{d}{dq^2} \Pi_{ZZ}(q^2) \Big|_{q^2=M_Z^2}.$$

Moreover, define a running wave function renormalization constant $Z_{Z^\star}(q^2)$ by

$$\frac{e_\star^2}{s_\star^2 c_\star^2} Z_{Z^\star} \equiv \frac{e^2}{s_\theta^2 c_\theta^2} Z_Z. \quad (56)$$

Calculating, we find

$$\begin{aligned} Z_{Z^\star}(q^2) &= Z_Z \left[1 - \pi'_{\gamma\gamma}(q^2) - \frac{c_\theta^2 - s_\theta^2}{c_\theta s_\theta} \Pi'_{\gamma Z}(q^2) \right] \\ &\simeq 1 + \frac{d}{dq^2} \Pi_{ZZ}(q^2) \Big|_{q^2=M_Z^2} - \Pi'_{\gamma\gamma}(q^2) - \frac{c_\theta^2 - s_\theta^2}{s_\theta c_\theta} \Pi'_{\gamma Z}(q^2). \end{aligned}$$

This makes

$$M_{NC} = e_\star^2 Q \left(-\frac{i}{q^2} \right) Q' + \frac{e_\star^2}{c_\star^2 s_\star^2} (T_3 - Q s_\star^2) \frac{Z_{Z^\star}}{q^2 - M_{Z^\star}^2} (T'_3 - Q' s_\star^2) \quad (57)$$

which is just the tree-level formula except that all bare quantities are replaced by starred renormalized quantities.

Exactly similar considerations can be followed for CC -induced scattering. Now one has the situation of Fig. 20.

Fig. 20. CC scattering with 1-loop oblique corrections.

Thus

$$M_{CC} = \frac{e_\star^2}{2s_\star^2} I_+ \frac{Z_{W^\star}}{q^2 - M_{W^\star}^2} I_-, \quad (58)$$

where

$$\frac{e_\star^2}{s_\star^2} Z_{W^\star} = \frac{e^2}{s_\theta^2} Z_W.$$

Hence the renormalized version of (35) is

$$A_{FB}^{\mu\mu}(M_Z^2) = \frac{3[1 - 4s_\star^2(M_Z^2)]^2}{[1 + \{1 - 4s_\star^2(M_Z^2)\}^2]^2}. \quad (59)$$

We have to consider M_{CC} and M_{NC} at $q^2 = 0$. After incorporating 1-loop oblique corrections, one has an effective Lagrangian density

$$\mathcal{L}_{EFF}^{\text{Weak}} = 4 \frac{G_\mu}{\sqrt{2}} [J_{\nu L}^+ J_L^{\nu-} + \rho (J_{\nu L}^3 - s_\star^2(0) J_\nu^Q) (J_L^{\nu 3} - s_\star^2(0) J^{\nu Q})],$$

with ρ being the ratio of the nonelectrodynamical part of M_{NC} to the corresponding M_{CC} at $q^2 = 0$. Thus, with this $\mathcal{L}_{EFF}^{\text{Weak}}$ for instance,

$$\Gamma_Z = \frac{Z_{Z^\star}(M_Z^2) \alpha_\star(M_Z^2)}{6s_\star^2(M_Z^2) c_\star^2(M_Z^2)} \sum_f [T_{3f} - s_\star^2(M_Z^2) Q_f]^2 N_f^C,$$

where, after QCD corrections,

$$N_\ell^C = 1 + \frac{3\alpha_{EM}(M_Z)}{4\pi} Q_\ell^2,$$

$$N_q^C = \left[1 + \frac{3\alpha_{EM}(M_Z)}{4\pi} Q_q^2 \right] \left[1 - \frac{\alpha_S(M_Z)}{\pi} + 0(\alpha_S^2) \right].$$

At $q^2 = 0$,

$$q^2 - M_{Z^*}^2 \rightarrow -M_{Z^*}^2 - \Pi_{ZZ}(0) = -\frac{e^2}{s_\theta^2 c_\theta^2} \left[\frac{v^2}{4} + \Pi_{33}(0) \right],$$

$$q^2 - M_{W^*}^2 \rightarrow -\frac{e^2}{2s_\theta^2} \left[\frac{v^2}{4} + \Pi_{11}(0) \right].$$

Finally,

$$M_{NC}^{NQED}(q^2 = 0) = [T_3 - s_\star^2(0)Q] \left[\frac{v^2}{4} + \Pi_{33}(0) \right]^{-1} [T_3 - s_\star^2(0)Q],$$

$$M_{CC}(q^2 = 0) = \frac{1}{2}T_+ \left[\frac{v^2}{4} + \Pi_{11}(0) \right]^{-1} T_-.$$

Excluding the group theory factors,

$$\rho = \left[\frac{v^2}{4} + \Pi_{11}(0) \right] \left[\frac{v^2}{4} + \Pi_{33}(0) \right]^{-1} \simeq 1 + \frac{4}{v^2} [\Pi_{11}(0) - \Pi_{33}(0)]$$

to 1-loop.

We can now consider the $Zf\bar{f}$ vertex. Since the outside coupling is fixed to be G_μ , as in (16), for the renormalized couplings we need to take

$$v_f \rightarrow \sqrt{\rho}[T_3 - Q_f s_\star^2(q^2)], \quad (60a)$$

$$a_f \rightarrow \sqrt{\rho}T_3, \quad (60b)$$

$$\rho = 1 + \frac{4\pi\alpha_{EM}}{s_\theta^2 c_\theta^2 M_Z^2} [\Pi_{11}(0) - \Pi_{33}(0)]. \quad (60c)$$

Now we need only specify $s_\star^2(q^2)$.

$\sin^2 \theta_W$

We discuss the “renormalization” of the sine of the Weinberg angle $\sin \theta_W$. The tree level s_θ is no longer an operative parameter and we need a definition of $\sin \theta_W$ via a renormalized physical process. Veltman and Passarino like $\sin^2 \theta_W \equiv 1 - M_W^2 M_Z^{-2}$, but the disadvantage there is that M_W not well-measured as yet. Another approach is to define $\sin^2 \theta_W$ as the ratio of coupling constants renormalized by the \overline{MS} scheme as in QCD, but the problem here is that it cannot be simply related to physical observables. We want to generalize the tree-level relation (15) and *define*

$$\sin(2\theta_W)|_Z \equiv \left(\frac{4\pi\alpha_*(M_Z^2)}{\sqrt{2}G_\mu M_Z^2} \right)^{1/2}, \quad (61)$$

a definition which clearly relates $\sin^2 \theta_W$ to physically observable quantities.

One should first calculate $s_\star^2(q^2) - \sin^2 \theta_W|_Z$. Recall from (55) and (49d) that

$$s_\star^2 = \frac{g'^2}{g^2 + g'^2} - e^2[\Pi'_{3Q}(q^2) - s_\theta^2 \Pi'_{QQ}(q^2)]. \quad (62)$$

Now (51) – (53) can be rewritten as

$$\frac{\delta\alpha_{\star EM}(M_Z^2)}{\alpha_{EM}} \simeq e^2 \Pi'_{QQ}(M_Z^2). \quad (63)$$

Furthermore, the import of (44) is that – in the oblique approximation –

$$G_{\mu\star}(0) \simeq G_\mu \left(1 - \frac{\Pi_{WW}(0)}{M_W^2} \right).$$

Thus

$$\frac{\delta G_{\mu\star}}{G_\mu} \simeq -\frac{e^2}{s_\theta^2} \Pi_{11}(0). \quad (64)$$

Again, from (36),

$$\begin{aligned} M_Z^2 &= M_{Zb}^2 \left[1 + \frac{\Pi_{ZZ}(M_Z^2)}{M_{Zb}^2} \right] \\ &= \frac{1}{4}(g^2 + g'^2)v^2 \left[1 + \frac{e^2}{s_\theta^2 c_\theta^2 M_Z^2} \left\{ \Pi_{33}(M_Z^2) - 2s_\theta^2 \Pi_{3Q}(M_Z^2) + s_\theta^4 \Pi_{QQ}(M_Z^2) \right\} \right]. \end{aligned}$$

Thus

$$\begin{aligned}\delta(\sin^2 \theta_W) = 2 \sin \theta_W \cos \theta_W \delta \theta_W &= \frac{\sin \theta_W \cos \theta_W \delta(\sin 2\theta_W)}{\cos^2 \theta_W - \sin^2 \theta_W} \\ &= \frac{2s_\theta^2 c_\theta^2}{c_\theta^2 - s_\theta^2} \frac{\delta(\sin 2\theta_W)}{\sin 2\theta_W}.\end{aligned}\quad (65)$$

Return to (15) and write it as

$$\sin 2\theta_{Wb} = \left(\frac{4\pi\alpha_{EMb}}{\sqrt{2}G_{\mu b}M_{Zb}} \right)^{1/2}, \quad (66)$$

$$\delta(\sin 2\theta_{Wb}) = \frac{1}{2} \left(\frac{4\pi\alpha_{EMb}}{\sqrt{2}G_{\mu b}M_{Zb}^2} \right)^{1/2} \left[\frac{\delta\alpha_{EM}}{\alpha_{EM}} - \frac{\delta G_\mu}{G_\mu} - \frac{\delta M_Z^2}{M_Z^2} \right]. \quad (67)$$

Now, by using (61) to (64), we can express $\sin^2 \theta_W|_Z$ as

$$\begin{aligned}\sin^2 \theta_W|_Z &= \sin^2 \theta_{Wb} + \delta(\sin^2 \theta_{Wb}) \\ &= \sin^2 \theta_{Wb} + \frac{s_\theta^2 c_\theta^2}{c_\theta^2 - s_\theta^2} \left[e^2 \Pi'_{QQ}(0) + \frac{e^2}{s_\theta^2 M_W^2} \Pi_{11}(0) \right. \\ &\quad \left. - \frac{e^2}{s_\theta^2 c_\theta^2 M_Z^2} \Pi_{33}(M_Z^2) - 2s_\theta \Pi_{3Q}(M_Z^2) + s_\theta^4 \Pi_{QQ}(M_Z^2) \right].\end{aligned}\quad (68)$$

Use (62) and (67) to write

$$\begin{aligned}s_\star^2(q^2) - \sin^2 \theta_W|_Z &= \frac{e^2}{c_\theta^2 - s_\theta^2} \left[\frac{\Pi_{33}(M_Z^2) - 2s_\theta^2 \Pi_{3Q}(M_Z^2) - \Pi_{11}(0)}{M_Z^2} \right. \\ &\quad \left. - (c_\theta^2 - s_\theta^2) \Pi'_{3Q}(q^2) \right] \\ &\quad + \frac{e^2 s_\theta^2}{c_\theta^2 - s_\theta^2} \left[s_\theta^2 \Pi'_{QQ}(M_Z^2) - c_\theta^2 \Pi'_{QQ}(0) \right. \\ &\quad \left. + (c_\theta^2 - s_\theta^2) \Pi'_{QQ}(q^2) \right].\end{aligned}\quad (69)$$

W-mass renormalization

We have already seen that

$$M_W^2 = M_{Wb}^2 + \Pi_{WW}(M_W^2) = M_{Wb}^2 + \frac{e^2}{s_\theta^2} \Pi_{11}(M_W^2). \quad (70)$$

Furthermore,

$$M_{Wb}^2 = M_{Zb}^2 \cos^2 \theta_{Wb}. \quad (71)$$

Using (68), (70) and (71)

$$\begin{aligned} M_W^2 = M_Z^2 \cos^2 \theta_W|_Z - \frac{e^2 c_\theta^2}{s_\theta^2 (c_\theta^2 - s_\theta^2)} & \left[\Pi_{33}(M_Z^2) - 2s_\theta^2 \Pi_{3Q}(M_Z^2) \right. \\ & \left. - \frac{s_\theta^2}{c_\theta^2} \Pi_{11}(0) - \frac{c_\theta^2 s_\theta^2}{c_\theta^2} \Pi_{11}(M_W^2) \right]. \end{aligned} \quad (72)$$

• Introduction to Oblique Parameters

Within the framework of the obliqueness approximation, the three oblique parametes [16] can be defined [18] as linear combinations of Π -functions, defined at $q^2 = 0$ and $q^2 = M_Z^2$. We first introduce the hypercharge current J_μ^Y as a linear combination of the electromagnetic current and the third weak isospin current:

$$J_\mu^Q = J_\mu^3 + \frac{1}{2} J_\mu^Y. \quad (73)$$

Now we define

$$\begin{aligned} S & \equiv \frac{16\pi}{M_Z^2} [\Pi_{33}(M_Z^2) - \Pi_{33}(0) - \Pi_{3Q}(M_Z^2)] \\ & = \frac{8\pi}{M_Z^2} [\Pi_{3Y}(0) - \Pi_{3Y}(M_Z^2)], \end{aligned} \quad (74a)$$

$$T \equiv \frac{4\pi}{s_\theta^2 c_\theta^2} M_Z^{-2} [\Pi_{11}(0) - \Pi_{33}(0)], \quad (74b)$$

$$U \equiv \frac{16\pi}{M_W^2} [\Pi_{11}(M_W^2) - \Pi_{11}(0)] - \frac{16\pi}{M_Z^2} [\Pi_{33}(M_Z^2) - \Pi_{33}(0)]. \quad (74c)$$

There are other definitions [19] of these parameters. Particularly popular is [20] the set $\epsilon_1, \epsilon_2, \epsilon_3$ where

$$\epsilon_1 = \alpha_{EM}T, \quad (75a)$$

$$\epsilon_2 = -\frac{\alpha_{EM}}{4s_\theta^2}U, \quad (75b)$$

$$\epsilon_3 = \frac{\alpha_{EM}}{4s_\theta^2}S. \quad (75c)$$

More general definitions of these parameters, going outside the obliqueness approximation, also exist [21].

1. There are two important aspects of the oblique parameters which should be highlighted. T and U receive nonzero contributions from the violation of weak isospin and are finite on account of the weak isospin symmetric nature of the divergence terms. S originates from the mixing between the weak hypercharge and the third component of weak isospin as a consequence of the spontaneous symmetry breakdown mechanism. Only soft operators (i.e. those with scale dimensionality less than four) are involved in the latter process. By Symanzik's theorem [22], these do not contribute to the leading divergences and therefore S is free of them. Furthermore, the nonleading divergences cancel out in the difference between $\Pi_{3Y}(M_Z^2)$ and $\Pi_{3Y}(0)$ leaving a finite S .

2. The LHS of (48) can be rewritten by inserting a complete set of states as

$$(2\pi)^4 \sum_n \delta^{(4)}(q - p_n) \langle \Omega | J_\mu^A(0) | n \rangle \langle n | J_\nu^B(0) | \Omega \rangle.$$

Any new physics effect from beyond SM would come from a new set of states and hence would be linearly adding to that from SM , i.e. the Π_{AB} functions receive contributions from different sources additively. This enables one to define $\tilde{\Pi}_{AB} = \Pi_{AB} - \Pi_{AB}^{SM}$. Of course, Π_{AB}^{SM} depends on the yet unknown top and Higgs masses quadratically and logarithmically in respective order, the latter being a consequence of Veltman's screening theorem [23].

The relationship between the oblique parameters and observables can be obtained by rewriting the Π -functions in terms of S, T and U in Eqs. (60) – (72). Specifically, (60c) changes to

$$\rho = 1 + \alpha_{EM}T. \quad (76)$$

Furthermore, (69) changes to (with $q^2 = M_Z^2$)

$$s_\star^2(M_Z^2) - \sin^2 \theta_W|_Z = \frac{\alpha_{EM}}{c_\theta^2 - s_\theta^2} \left(\frac{1}{4} S - s_\theta^2 c_\theta^2 T \right) \quad (77)$$

and (72) changes to

$$M_W^2 = M_Z^2 \cos^2 \theta_W|_Z + \frac{\alpha_{EM} M_Z^2 c_\theta^2}{c_\theta^2 - s_\theta^2} \left(-\frac{1}{2} S + c_\theta^2 T + \frac{c_\theta^2 - s_\theta^2}{4s_\theta^2} U \right). \quad (78)$$

Moreover, one can split

$$(S, T, U) = (S, T, U)^{SM} + (\tilde{S}, \tilde{T}, \tilde{U})$$

and consequently rewrite (76) – (78) as

$$\rho = \rho^{SM} + \alpha \tilde{T}, \quad (79)$$

$$s_\star^2(M_Z^2) = [s_\theta^2(M_Z^2)]^{SM} + \frac{\alpha_{EM}}{4(c_\theta^2 - s_\theta^2)} (\tilde{S} - 4c_\theta^2 s_\theta^2 \tilde{T}), \quad (80)$$

$$M_W^2 = (M_W^2)^{SM} + \frac{\alpha_{EM} M_Z^2 c_\theta^2}{4s_\theta^2(c_\theta^2 - s_\theta^2)} [4c_\theta^2 s_\theta^2 \tilde{T} - 2s_\theta^2 \tilde{S} + (c_\theta^2 - s_\theta^2) \tilde{U}]. \quad (81)$$

Here ρ^{SM} , $[s_\star^2(M_Z^2)]^{SM}$ and $(M_W^2)^{SM}$ are these quantities, calculated to 1-loop in the standard model in terms of α_{EM} , G_μ and M_Z as well as fermionic and Higgs masses.

The determination of \tilde{S} and \tilde{T} from experiment is best done as follows. One calculates the differential cross section for the process $e^+e^- \rightarrow f\bar{f}$ in the Z lineshape region with the \star -scheme effective Lagrangian and the couplings v_f , a_f of (60 *a, b*). The latter are rewritten in terms of \tilde{S} and \tilde{T} which are obtained by detailed fits with the millions of accumulated data points. The latest fit with 5 million data points yields [24], for $m_t = 160$ GeV and $m_H = 100$ GeV,

$$\begin{aligned} \tilde{S} &= -0.49 \pm 0.31 \\ \tilde{T} &= -0.10 \pm 0.32. \end{aligned} \quad (82)$$

Thus \tilde{T} is compatible with the null value of SM whereas in \tilde{S} there is hint of a nonzero value at the 1.5σ level. The variations of these numbers with changes in m_t and m_H have also been studied [24]. In particular \tilde{S} is insensitive to

variations in m_t in the range of interests. The extraction of \tilde{U} is rather inaccurate because of its sensitive dependence [vide (81)] on the W -mass which is rather poorly known. Using $M_W = 80.24 \pm 0.10$ GeV and the \tilde{S}, \tilde{T} values of (82), one is led to

$$\tilde{U} = -0.11 \pm 0.82. \quad (83)$$

The error will be significantly reduced once the W -mass is better known.

The oblique parameters are powerful probes for certain types of new physics. Any scenario which goes beyond SM will have particles heavier than those in the latter. A most important question [25] is how the effects of such particles on SM processes would act as their masses are made larger and larger. If these mass terms are $SU(2)_L \times U(1)_Y$ gauge-invariant, those effects decouple as inverse powers of the heavy masses and the new physics is of a decoupling type. An example of this is the supersymmetric extension of the SM . Contrariwise, for mass terms that are gauge-variant vis-a-vis $SU(2)_L \times U(1)$ transformations, those effects do not decouple even as the heavy masses become larger. This type of nondecoupling new physics is caused by extra heavy chiral fermion generations or condensate models such as technicolor.

In general, a decoupling type of new physics leads to rather small (compared to unity) values of $\tilde{S}, \tilde{T}, \tilde{U}$ for the new mass-scale in the sub-TeV to TeV region. Thus knowledge of the oblique parameters (with the kind of accuracy that is realistically feasible) cannot significantly test or constrain such models. This is, however, not the case with nondecoupling new scenarios. In particular, \tilde{S} is a rather sensitive probe for chiral fermion condensate models. Specifically, most technicolor and extended technicolor scenarios predict [26] large positive $S \geq 0.4$ and are disfavored by the data.

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